

LINEAR CIRCUIT ANALYSIS | **01**
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INTRODUCTION

All the branches of Electrical Engineering are based on two fundamental theories: Electric Circuit Theory and Electromagnetic Theory.

The well known branches of Electrical Engineering: power, machines, control, electronics, communication and instrumentation, are based on Electric Circuit Theory.

The basic Electric Circuit Theory is an important course for beginning student in Electrical Engineering.

This course is also valuable for physical sciences: Energy Systems, Applied Mathematics and Physics etc.

INTRODUCTION

Most often in Electrical Engineering, the energy is transferred from one point to another.

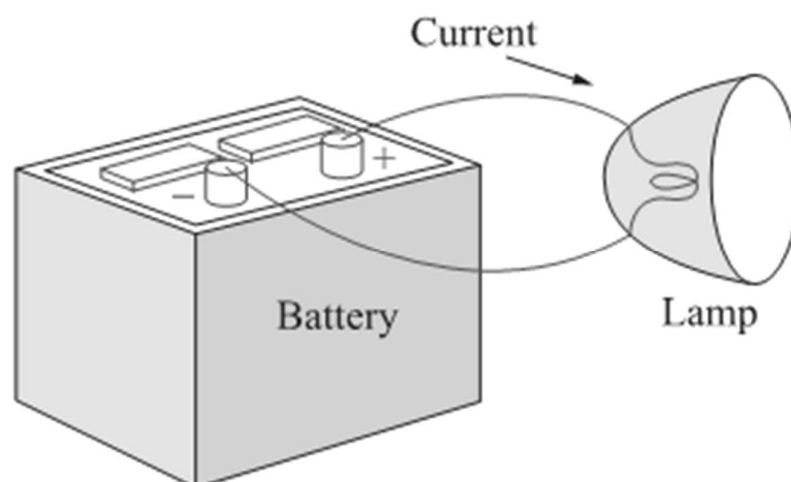
To do this requires an interconnection of electrical devices.

Such interconnection is referred to as an Electric Circuit, and each component of the circuit is known as an Element.

An Electric Circuit is an interconnection of electrical elements.

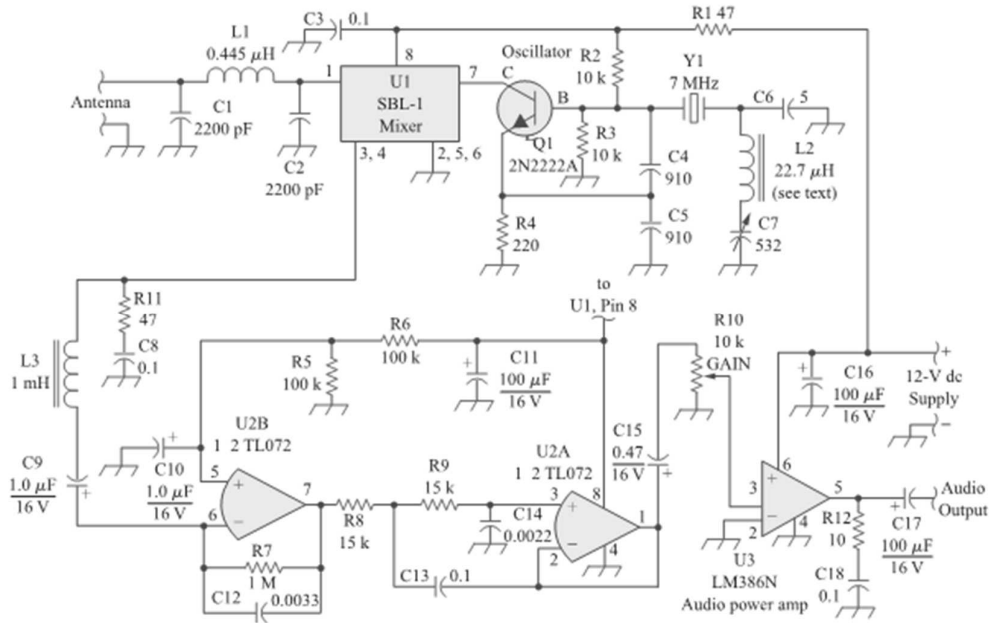
ELECTRIC CIRCUIT

A simple Electric Circuit



ELECTRIC CIRCUIT

Electric Circuit of Radio Receiver



ELECTRIC CIRCUIT

Electric Circuits are used in numerous electrical systems to accomplish different tasks.

Our objective is to analyze different electric circuits.

By the analysis of a circuit, we mean a study of the behavior of the circuit: How does it respond to a given input. How do the interconnected elements and devices in the circuit interact.

SYSTEMS OF UNITS

According to International System of Units (SI), there are six principal units from which the units of all physical quantities can be derived.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

SYSTEMS OF UNITS

One great advantage of SI units is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

CHARGE

The most basic quantity in an electric circuit is the Electric Charge.

Charge is an electrical property of the atomic particles of which matter consists, measured in Coulombs (C).

All matter is made of atoms and each atom consists of electrons, protons and neutrons.

The charge on an electron is negative and equal in magnitude to 1.602×10^{-19} C.

The proton carries the positive charge of the same magnitude as the electron.

CHARGE

The Coulomb is a large unit for charges. In 1 C of charge, there are 6.24×10^{18} electrons.

In nature, the charges occur in integral multiples of electronic charge, $e = -1.602 \times 10^{-19}$ C.

The law of conservation of charge states that charge can neither be created nor destroyed, only transferred.

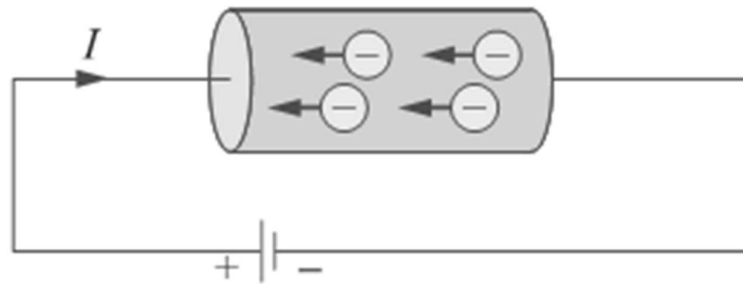
A unique feature of charge is the fact that it is mobile; it can be transferred from one place to another, where it can be converted to another form of energy.

CURRENT

The motion of charges creates electric current.

It is conventional to take the current flow as the movement of positive charges.

That is opposite to flow of negative charges.



CURRENT

Electric Current is the time rate of change of charge, measured in amperes (A).

1 ampere = 1 coulomb/second

Mathematically,

$$i \triangleq \frac{dq}{dt}$$

Charge transferred,

$$Q \triangleq \int_{t_0}^t i dt$$

DIRECT CURRENT (DC)

A Direct Current (DC) is a current that remains constant with time.

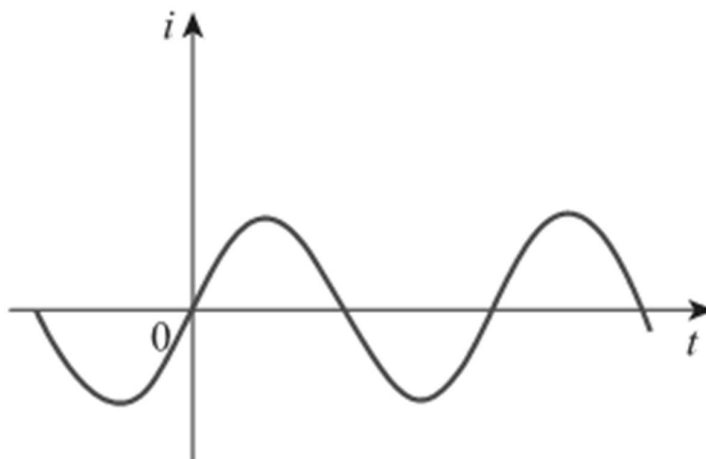
Conventionally, the symbol I is used to represent constant current.



ALTERNATING CURRENT (AC)

An Alternating Current (AC) is a current that varies sinusoidally with time.

Conventionally, the symbol i is used to represent time-varying current.



PROBLEMS

How much charge is represented by 4600 electrons?

$$-7.369 \times 10^{-16} \text{ C}$$

Calculate the amount of charge represented by 4 million protons?

$$6.408 \times 10^{-13} \text{ C}$$

The total charge entering a terminal is $q = 5t \sin 4\pi t$ mC. Calculate current at $t = 0.5$ s?

$$31.42 \text{ mA}$$

The total charge entering a terminal is $q = (10 - 10e^{-2t})$ mC. Calculate current at $t = 0.5$ s?

$$7.36 \text{ mA}$$

PROBLEMS

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s, if current passing the terminal is $i = (3t^2 - t)$ A?

$$5.5 \text{ C}$$

The current flowing through an element is:

$$i = \begin{cases} 2 \text{ A}, & 0 < t < 1 \\ 2t^2 \text{ A}, & t > 1 \end{cases}$$

Calculate the charge entering the element from $t = 0$ s to $t = 2$ s?

$$6.667 \text{ C}$$

VOLTAGE

To move an electron in a conductor requires some work or energy transfer.

This work is performed by an external electromotive force (emf), represented as battery.

This emf is also known as Potential Difference or Voltage.

Voltage is the energy required to move a unit charge through an element, measured in volts (V).

$$1 \text{ volt} = 1 \text{ joule/coulomb}$$

VOLTAGE

The voltage v_{ab} between two points a and b in an electric circuit is the energy or work needed to move a unit charge from a to b .

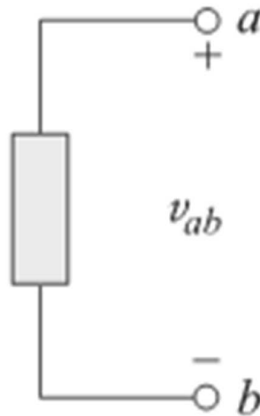
Mathematically,

$$v_{ab} \triangleq \frac{dw}{dq}$$

Voltage is always specified by its polarity and exists across an element.

VOLTAGE

The v_{ab} can be interpreted in two ways: (1) point a is at potential of v_{ab} volts higher than point b , or (2) the potential at point a with respect to b is v_{ab} .



DC VOLTAGE

A DC Voltage is a voltage, the magnitude and polarity of which remains constant with time.

In DC voltage, the two points are considered as positive and negative terminals.

The positive terminal is represented as higher potential terminal.

The negative terminal is represented as lower potential terminal.

AC VOLTAGE

An AC Voltage is a voltage that varies sinusoidally with time..

In AC voltage, the two points are considered as phase and neutral terminals.

The phase terminal is represented as higher potential terminal.

The neutral terminal is represented as lower potential terminal.

POWER

Power is time rate of expending or absorbing energy, measured in watts (W).

Mathematically,

$$p \triangleq \frac{dw}{dt}$$

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$

Instantaneous

Power,

$$p = vi$$

POWER

The power absorbed and supplied by an element is the product of the voltage across the element and current through it.

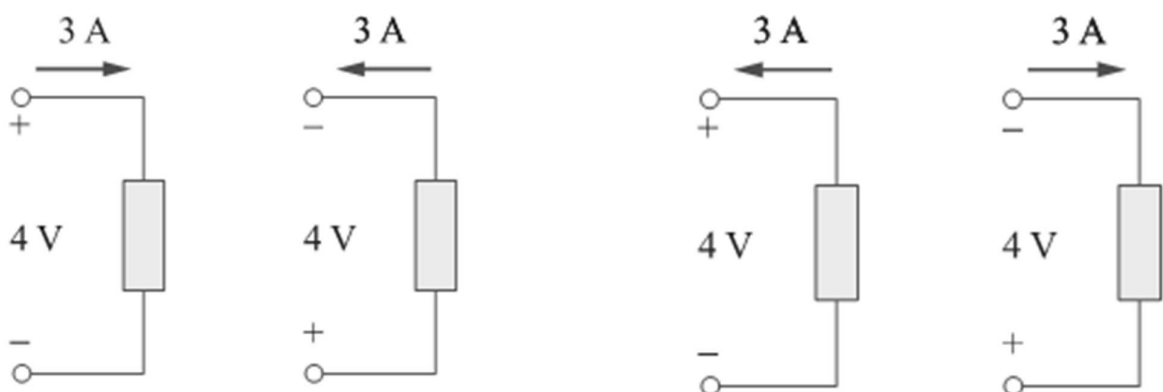
To specify the sign of power, passive sign convention is used.

Passive Sign Convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

POWER

If the power has positive sign, power is being delivered to or absorbed by the element.

If the power has negative sign, power is being supplied by the element.



POWER

The law of conservation of energy must be obeyed in any electric circuit.

The algebraic sum of power in a circuit, at any instant of time, must be zero.

$$\sum p = 0$$

The total power supplied to the circuit must balance the total power absorbed.

$$+\text{Power absorbed} = -\text{Power supplied}$$

ENERGY

Energy is defined as the capacity to do work, measured in joules (J).

Mathematically,

$$w = \int_{t_0}^t p \, dt = \int_{t_0}^t vi \, dt$$

The electric power utility companies measure energy in watt-hours (Wh).

$$1 \text{ Wh} = 3,600 \text{ J}$$

PROBLEMS

An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

20 C, 115 V

To move charge q from point a to point b requires -30 J. Find the voltage drop v_{ab} if: (a) $q = 2$ C, (b) $q = -6$ C.

-15 V, 5 V

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3 di/dt$.

53.48 W, -6.396 kW

PROBLEMS

How much energy does a 100-W electric bulb consume in two hours?

720 kJ, 200 Wh

A stove element draws 15 A when connected to a 240-V line. How long does it take to consume 60 kJ?

16.667 s

CIRCUIT ELEMENTS

Element is the basic building block of a circuit.

An electric circuit is simply an interconnection of elements.

Circuit Analysis is the process of determining the voltages across (or the currents through) the elements of the circuit.

There are two types of elements found in electric circuits: passive elements and active elements.

PASSIVE ELEMENTS

A Passive Element is that which is not capable of generating energy, rather it absorbs energy.

Examples of the passive elements are resistors, inductor and capacitors.

ACTIVE ELEMENTS

An Active Element is that which is capable of generating energy.

Typical active elements include generators, batteries and operational amplifiers.

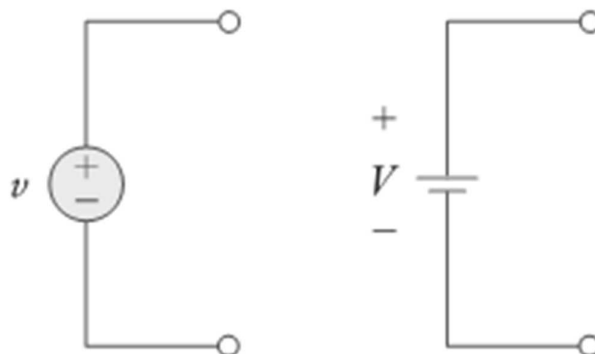
The most important active elements are voltage or current sources that generally deliver power to circuits connected to them.

There are two kinds of sources: independent and dependent sources.

INDEPENDENT SOURCE

An ideal Independent Source is an active element that provides voltage or current that is completely independent of other circuit elements.

An ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage.



INDEPENDENT SOURCE

Physical sources such as generators and batteries may be regarded as approximations to ideal voltage sources.

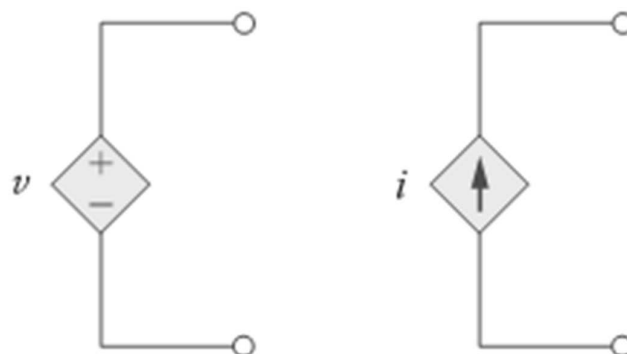
An ideal independent current source delivers to the circuit whatever voltage is necessary to maintain the designated current.



DEPENDENT SOURCE

An ideal Dependent (or Controlled) Source is an active element in which the source quantity is controlled by another voltage or current.

The dependent sources are usually designated by diamond shaped symbols.



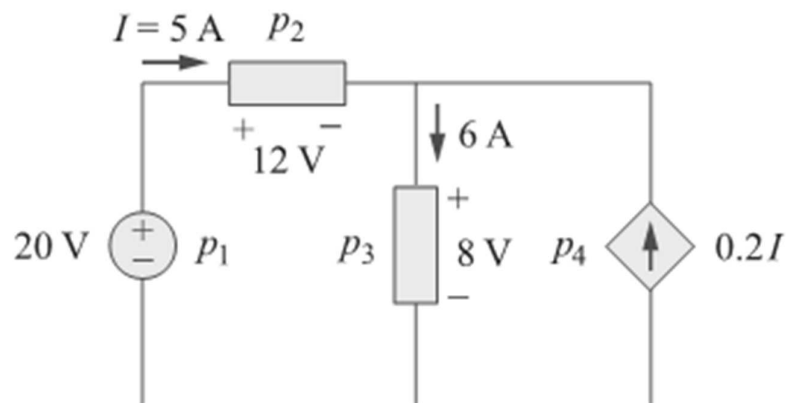
DEPENDENT SOURCE

There are four possible types of dependent sources as following;

1. A voltage-controlled voltage source (VCVS)
2. A current-controlled voltage source (CCVS)
3. A voltage-controlled current source (VCCS)
4. A current-controlled current source (CCCS)

PROBLEMS

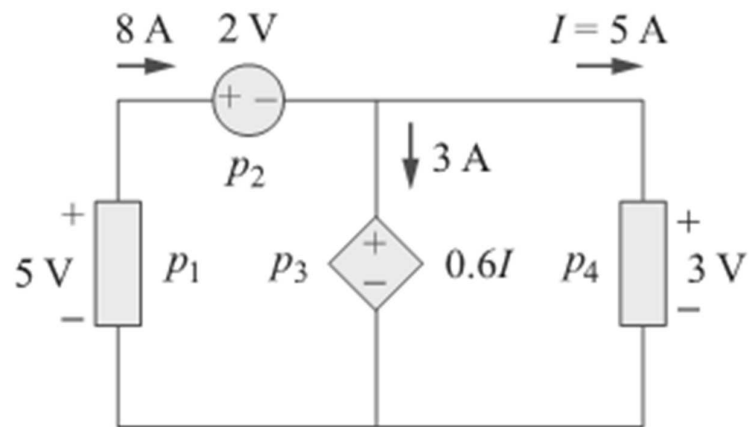
Calculate the power of each element?



-100 W, 60 W, 48 W, -8 W

PROBLEMS

Calculate the power of each element?



-40 W, 16 W, 9 W, 15 W

REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 01 – Basic Concepts (1.1 – 1.6)

Exercise Problems: 1.1 – 1.20

Do exercise problem yourself.